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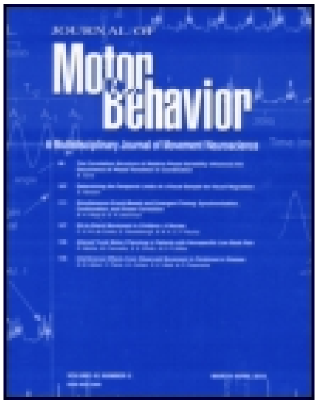
Peter J Beek
Vrije Universiteit Amsterdam



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P. J. Beek [a] [b] & A. A. M. van Santvoord [c]
[a] Faculty of Human Movement Sciences , Vrije Universiteit of Amsterdam , USA
[b] Center for the Ecological Study of Perception and Action , University of Connecticut , Storrs, USA
[c] Faculty of Human Movement Sciences , Vrije Universiteit of Amsterdam
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P. J. Beek
Faculty of Human Movement Sciences
Vrije Universiteit of Amsterdam
and Center for the Ecological Study of Perception and Action,
University of Connecticut, Storrs

A. A. M. van Santvoord
Faculty of Human Movement Sciences
Vrije Universiteit of Amsterdam

ABSTRACT. How beginning jugglers discover the temporal constraints governing the juggling workspace while learning to juggle three balls in a cascade pattern was the subject of this investigation. On the basis of previous theoretical and experimental work on expert jugglers, we proposed a three-stage model of the learning process, for which objective evidence was sought. The first stage consists of learning to accommodate the real-time requirements of juggling, as expressed in Shannon's equation of juggling, which states that, averaged over time, the cycle time of the hands should be a fixed proportion of the cycle time of the balls. The second stage of learning consists of discovering the primary frequency lock of .75 between the shorter term dynamical regime underlying the repetitive subtask of transporting a ball and the longer term dynamical regime underlying the total hand loop cycle. The third and last stage of learning consists of discovering the principles of frequency modulation from .75 to lower (averaged) values of the proportion of time that a hand carries a ball during the total hand cycle time. Twenty subjects were taught to juggle three balls in a cascade pattern. Ten subjects were trained with the aid of an instructor and a metronome, and 10 with the instructor only. The metronome proved to be of no particular additional help, but the timing results obtained were in agreement with the proposed three stages of learning. The picture that emerged from this study was that learning a new motor skill involves the discovery of invariances or fixed points in the perceptual-motor workspace associated with that skill, from which excursions can be made and the skill further refined. Because these fixed points afford stability of operation, discovering them logically and factually precedes the acquisition of the functional adaptability and flexibility of operation ("flair") inherent to frequency modulation.

Key words: dynamical systems approach, juggling, motor learning, motor skill acquisition, nonlinear dynamics, relative timing.

How the ability to successfully coordinate and control new movements is acquired is one of the most challenging problems in movement science. To date, there is neither a full-fledged theoretical framework nor an agreed-upon experimental methodology to approach this problem. Currently, however, there are some interesting new developments in this area, inspired by dynamical systems theory, as testified to by the articles collected in this single-theme issue. After about a decade of investigating the formative principles at work in the context of movement coordination and control with the tools and concepts of nonlinear dynamics, the prospects of a dynamical theory of motor skill acquisition have become real (Beek, 1989a, 1989b; Fowler & Turvey, 1978; Haken, Kelso, & Bunz, 1985; Kelso, 1984; Kugler, Kelso, & Turvey, 1982; Saltzman & Kelso, 1987).

From a dynamical systems perspective, learning a new motor skill is generally viewed as the transition from one particular dynamical state to another, which confronts the researcher with the problem of identifying (a) the (sometimes "fuzzy") dynamics of the initial movement pattern, (b) the attractor state of the movement pattern toward which the system evolves, and (c) the dynamical principles that govern the transition. Roughly speaking, two dynamical approaches to this problem

Correspondence concerning this paper should be addressed to Peter J. Beek, Department of Psychology, Faculty of Human Movement Sciences, Vrije Universiteit of Amsterdam, Van der Boechorststraat 9, 1081 BT Amsterdam, The Netherlands.

area may be distinguished, each with its own merits and drawbacks. The first employs concepts from synergetics in studying learning and recall (Schöner, 1990; Schöner & Kelso, 1988a, 1988b; Zanone & Kelso, in press). Synergetics provides an interdisciplinary, strictly operational conceptual framework for studying pattern formation in complex systems (Haken, 1977, 1983), that derives its unity from a set of generalized mathematical principles. The second approach, sometimes called the natural-physical approach (Kugler, 1986), is more hybrid in its choice of physical concepts and focuses on explorations of perceptual-motor workspaces (Newell, Kugler, van Emmerik, & McDonald, 1989).

The basic strategy adopted in the *synergetic* approach is first to identify the intrinsic dynamics of a particular task's execution, preferably one that involves two stable phase relations and a phase transition between them (such as in the well-known experiments on finger and hand coordination by Kelso and his colleagues; cf. Haken et al., 1985; Kelso, 1984). The next step is to endow the behavioral information specifying a to-be-learned movement pattern with dynamics, for example, the extrinsic dynamics, and to dynamically model the transition from the initial intrinsic dynamics to the intrinsic dynamics of the learned movement pattern in terms of competition and/or cooperation between the intrinsic and the extrinsic dynamics. By treating, for instance, a temporally structured environment (i.e., a metronome) as a dynamic acting on the intrinsic relative phase dynamics of finger movements, Schöner and Kelso (1988a, 1988b; Schöner, 1990) were able to show how these environmental forces cooperate or compete with the forces corresponding to the intrinsic dynamics in the transition to the required movement pattern. Recently, the predictions from this work have received experimental confirmation in a study by Zanone and Kelso (in press), who showed that learning may indeed be characterized as a nonequilibrium phase transition, namely, a qualitative change in the dynamics underlying coordination. Such endeavors address a restricted class of learning phenomena because the initial state cannot be "fuzzy" but needs to be well defined (e.g., by virtue of the bistability of the initial movement pattern), as is the case for the end state. Under these circumstances, the transition from the initial state to a new state can be studied rigorously.

The *natural-physical approach*, in contrast, has been concerned primarily with the search strategies employed by skill learners in exploring particular perceptual-motor workspaces. A problem confronted in this approach is that it requires, ideally, that the attractor layout over which the search takes place be known a priori, which is especially difficult because this attractor layout develops as a function of practice (i.e., of the exploratory behavior itself). To circumvent this problem of the nonstationarity of the workspace and to make the approach experimentally tractable, one may introduce a task criterion in the form of an externally defined task space potential, such as in the Krinskiy and Shik task studied by Newell et al. (1989; cf. Fowler & Turvey, 1978). In a sense, such an externally defined task dynamics is the exact opposite of the intrinsic dynamics of a task such as finger wiggling, in that the internal constraints of the human action system largely determine the latter and are irrelevant to the former. Typically, however, the attractor layout of the workspaces associated with complex motor actions involving tools or implements results from the interplay between organismic, environmental, and task constraints (Newell, 1986), and from none of these sources alone.

Both the synergetic strategy of identifying principles of learning and recall and the strategy of identifying the search strategies used in learning require that either the attractor layout of the initial and end state dynamics (intrinsic dynamics) or of the to-be-learned task dynamics (external dynamics) be known. In learning a new motor skill, however, the initial intrinsic dynamics are generally very difficult to assess as long as the skill in question cannot be performed, and the externally defined dynamics of the task criterion are usually opaque as well. Thus, as it stands now, the synergetic approach is confined to studying learning phenomena as order-order transitions only, whereas the natural-physical approach is restricted to studying exploration in externally defined workspaces that circumvent—but do not solve—the problem of the nonstationarity of the global workspace defined over actor and environment. A wide and arguably most interesting class of learning involves "disorder-order" transitions, in the sense that the workspace has to be set up first and gradually developed next. Nonstationarity of the workspace is an outstanding characteristic of this type of transitions.

To "get a handle" on these issues in motor skill learning, this article reports on research that pursued a third, alternative strategy. The strategy starts by identifying the dynamics of the end state toward which the learning process evolves rather than the intrinsic dynamics of the initial state or the dynamics associated with an externally defined target state. It is based on the conviction that precise knowledge of what is being learned, obtained from extensive study of expert performance, (ideally) precedes attempts to come to terms with questions as to how learning takes place. Once the laws and regularities of a well-mastered skill are known, a frame of reference becomes available in relation to which the changes in a learning process may be interpreted. Such an approach, however, has to be based on the assumption that the skill learner follows a path that leads continuously (in the technical, mathematical sense of integrability) to the dynamical end state of the very skillful. This assumption is valid if a neural network that relaxes to some end state (i.e., the expert state) can, in principle, be used to model the learning phenomena in question. It does not imply that saltatory phenomena or other discontinuities, such as transitions between coordinative styles, should be absent.

With these aims in mind, our efforts in recent years have been directed at uncovering the perceptual-motor workspace of cascade juggling (a figure-eight pattern involving an odd number of objects; see Figure 1) of expert performers, especially in terms of the temporal and spatial constraints governing the attractor layout of this workspace.

The advantage of studying a perceptual-motor workspace of a demanding skill such as cascade juggling is that this skill involves such severe temporal and spatial constraints that the task solutions that can be achieved by the juggler are small in number and can be described precisely. In the present article, we focus on the temporal constraints on juggling and seek a description of the process of learning the cascade juggle in terms of learning to accommodate these constraints.

Cascade juggling is a complex, cyclic activity that is intrinsically rhythmic. Because of the severe nature of the temporal constraints, we are, in this particular case, and contrary to many other rhythmic activities, able to define precisely what is meant by the term "intrinsically rhythmic": A pair of coordinated hands ($H = 2$) manipulate N balls such that, averaged over time, N^* of them are airborne. The act has to match the average flight time (t_f) of the ball so that it comprises N^*/H times the average cycle period of one hand (t_h). The constraint implies that, for cascade juggling to take place, there has to be a locking at the macroscopic level of task execution between the average frequency of the hand [$2\pi/t_h \equiv 2\pi/(t_l + t_u)$, where t_l is the average time that a hand is loaded with a ball and t_u is the average time that a hand moves empty, i.e., unloaded] and the average frequency of the balls [$2\pi/t_b \equiv 2\pi/(t_l + t_f)$], so that $t_b/t_h = N/H$ (Beek, 1989a, 1989b). In other words, the

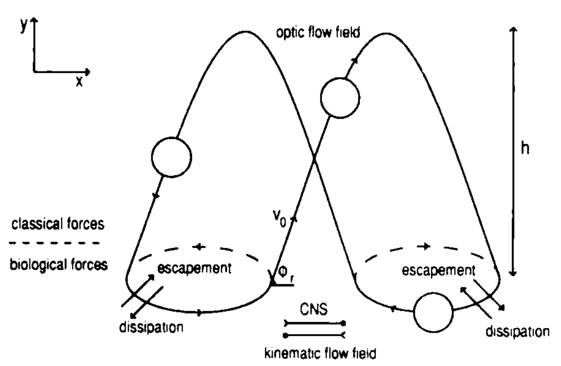

FIGURE 1. Schematic representation of juggling three balls in a figure-eight pattern, momentary situation ϕ_r is the angle of release; v_0 is the velocity of the ball at the moment of release; h is the height to which the balls are thrown relative to the point of release; CNS is the central nervous system.

juggling hands and juggled balls must satisfy a general timing requirement in which the ratio of hand cycle time to ball cycle time must, on average, equal the ratio of number of hands to number of balls. This timing constraint was identified first by Claude Shannon (cf. Horgan, 1990; Raibert, 1986). It applies to all juggling patterns and may, therefore, be called the common equation of juggling. It is the basic temporal constraint of juggling that has to be satisfied in order to perform a sustained act.

It has been argued by one of the authors (Beek, 1989a; Beek & Turvey, 1991) that, in addition to the common equation of juggling, a second, more microscopic mode-locking principle plays a role. This principle is expressed by the formula $W(t_l - t_u) = (t_l + t_u)$, where W is the winding number for the internal organization of a juggling hand cycle. This principle would result in mode locking (i.e., fixed proportionalities) between the shorter term (t_l) dynamical regime underlying the repetitive subtask of transporting a ball and the longer term ($t_l + t_u$) dynamical regime underlying the rhythmic movement of the hand, if W is an integer. This can be seen by taking $k \equiv t_l/(t_l + t_u)$ and rewriting $W(t_l - t_u) = (t_l + t_u)$ into $W\{k - (1 - k)\} = 1$ or $k = .5(1 + W^{-1})$. The importance of the latter expression is that its suggestion that $W = 2$ (leading to the value of $k = .75$) is the lowest possible integer value of W at which mode locking can be achieved, for $W = 1$ represents the physically impossible case of $k = 1$ (i.e., all hands occupied with balls at all times). It also suggests that $W = 2$ leads to the most stable act, because it is generally known from nonlinear dynamics that periodic behavior at winding numbers composed of small integers (i.e., 1:1, 1:2, 1:3) is more stable than at winding numbers composed of larger integers.

These suggestions were confirmed in experiments on expert cascade jugglers (Beek, 1989a, 1989b, in press), which revealed that in the case of juggling five and seven balls, the ratios of time taken up by carrying a ball to a hand's cycle time were indeed always close to .75, on average, regardless of the absolute frequency of juggling. In the case of juggling three balls, k had a tendency to be lower than .75: It varied between .54 and .86, with a mean of .70, and was, to some degree, a function of the absolute cycle frequency. These results suggest that $k = .75$ defines a preferred point in the dynamical workspace of cascade juggling but is not an obligatory property of juggling in the same way as the common equation of juggling, which has to be obeyed at all times. In juggling five and seven balls, $W = 2$ seems to be the only winding number at which mode locking can be achieved. For the less demanding act of juggling three balls, however, higher rational winding numbers may be attained by expert performers up to $W = 12$ ($k = .54$).

By regarding the value of .75 as the first and most stable frequency lock between the time a hand carries a ball and the total hand cycle time, we could define a measure of the degree of frequency modulation (i.e., quasiperiodicity) to which the individual k values in the three-ball experiments related linearly with an intercept of $k = .75$ (see the Appendix for a more elaborate mathematical model for frequency locking at, and frequency modulation around, the value of .75).

The picture emerging from this theoretical and empirical work is that (cascade) jugglers essentially juggle according to the same blueprint, even though this invariance may be obscured in the case of juggling three balls, in which the spatiotemporal liberties are such that considerable modulation, involving higher-order frequency locks, may take place in expert jugglers. Furthermore, the modulation of the component frequencies of juggling in the direction of k values smaller than .75 proves to be a skill in itself, as it is directly associated with a larger time-averaged number of airborne objects ($N^* = N - kH$).

In sum, the "motor problem" of juggling, to use Bernstein's (1967) apt term, is to a considerable extent a problem of appropriate timing (cf. Austin, 1976). Hence, in large part, the problem of learning to juggle is that of mastering these timing relations. From the identified principles of timing in juggling, the following hypothetical picture of the learning process of juggling emerges, involving three sequential stages in the evolution of the attractor layout: (a) accommodating the real-time requirements of the common equation of juggling, that is, achieving coarsely tuned ($N{:}H$) mode locking between the frequency of the balls and the frequency of the hands. During this stage, the k variable is expected to be different from .75 and not expected to change systematically; (b) accommodating the requirements for the internal partitioning of a juggling hand cycle at the lowest possible integer winding number ($W = 2$, $k = .75$), that is, achieving finely tuned mode locking between the time that a hand carries a ball and the total hand cycle; and (c) discovering and obeying the rules of frequency modulation, involving complex transients to higher-order mode locks of k, so that the time-averaged number of airborne balls is increased and room is created to juggle with flair (great flexibility). Thus, each stage is associated with specific task criteria that the juggler attempts to achieve.

In the present study we investigated whether these hypothetical stages of the skill acquisition process of juggling, in particular the key role assigned to the mode-locked solution of $k = .75$, would be reflected in timing data obtained during the learning process of 20 people who had never juggled before. To appreciate this point, it is important to realize that, as explained above, the mode-locked solution of $k = .75$ is not the only solution available. This study did not address such questions as how novices develop strategies to go from one stage of learning to the next or why one individual requires less time than another to find such strategies. Instead, we sought objective evidence for each learning stage identified above.

Novices were taught to execute the cascade pattern by a professional teacher. In addition to the instructor, 10 of them were allowed to make use of an auditory learning aid in the form of a metronome with whose fixed beep-beep interval the juggler could synchronize the throwing (or catching) actions of the hands. We expected that the metronome would be of help in stabilizing the macroscopic frequency of juggling by fixing the hand cycle time relative to the ball cycle time and thus facilitating the discovery of the mode-locked solution of $k = .75$ (even though the metronome beeps do not specify this value). After all, when a particular partitioning of an interval is called for, it may be helpful if that interval has a fixed, externally defined value.

Method

Subjects

Thirteen female and 7 male students participated in the experiment. At the onset of the experiment, none of them was able to juggle or had ever tried to learn how. Their average age was 21.1 years with a standard deviation of 3.3 years. The only knowledge they had of the experiment was that they were going to learn to cascade juggle with three balls (so-called *stage* balls: diameter 7.3 cm, weight 130 g).

Procedures

Over a period of 2 weeks, the subjects attended 10 practice sessions of 1/2 h each. Practice outside these sessions was not allowed. The subjects were instructed by a professional juggler who was also an experienced teacher. At the beginning of the first session, the novices were instructed to learn the cascade in a stepwise fashion, according to the following instructions.

1. Take one ball and throw it from one hand to the other. Throw it from about waist height to a point level with the top of your head.
2. Hold one ball in each hand, throw the right-hand ball in an arc toward your left hand. As it peaks, throw the second ball in an arc underneath it toward the left hand. Catch the first ball in your left hand and the second in your right.
3. Hold two balls in your right hand and one in your left. Throw the first ball in your right hand toward your left one. As it peaks, throw the ball in the left hand toward the right. As the second ball peaks, throw the final ball from your right hand. Catch none of the balls.
4. Do as before, but now catch the balls and throw them as the previously thrown ball peaks. Keep on repeating the sequence, and you are juggling.

After these initial steps, the subjects attempted to sustain juggling as long as possible, while receiving individual-specific instructions from the teacher.

During all 10 sessions, the subjects themselves kept track of their improvements by counting the number of consecutive throws that resulted in a successful catch. They reported to the experimenters each time they accomplished an additional complete juggling cycle ($N \times H$ hand cycles), beginning with one complete juggling cycle (six consecutive throws and six consecutive catches), two complete cycles, three, and so on, up to six. These moments in time were registered by the experimenters, and the subjects were invited to juggle one complete juggling cycle in front of the camera after each criterion. These recordings will be referred to as the *cycle trials* and will be used to investigate the early phase of learning. The instructor and the two experimenters followed the progress being made by each subject as closely as possible, to check on the self-report method.

After three sessions, the subjects were divided into two equally skilled groups on the basis of the progress they had made, defined as the number of cycles the individual subjects were able to juggle at that moment in time, so that the two groups could be matched (pairwise) for initial learning rate.

During the following seven sessions, one group practiced with an auditory metronome present, and the other group without. At the beginning of a session, the metronome was set at a frequency of 1.66 Hz, which corresponds to a k value of .75 when the balls are thrown to a height of 1 m (relative to the waist-high throw position). During training, the subjects were encouraged to freely set the pace of the metronome to a desired value. The selected frequencies ranged between 1.5 and 1.8 Hz. The subjects were instructed to synchronize either their throwing or their catching actions with the beeps produced by the metronome.

All subjects were filmed at the beginning of the 4th session (pretest), during the 7th session (midtest), and at the end of the 10th session (posttest). On each test occasion, two complete juggling cycles, that is, 12 hand cycles (2 × 3 balls × 2 hands), were recorded, always without the metronome. These recordings will be referred to as the *test trials* and will be used to investigate the effect of the metronome. Multiple registrations (of shorter bouts of juggling) were made of those jugglers who could not accomplish this feat at the time of the pretest, so that we would end up with an equal number of observations for all subjects.

The juggling metronome. A program running on a personal computer produced the sound stimulus for the instructor-plus-metronome group. The program was so designed as to produce tones of 50-ms duration alternating on the left (220 Hz) and the right (275 Hz) channel, and with an interval time corresponding to the time between a throw (or catch) with one hand and the successive throw (or catch) with the other hand. The duration of the beep-beep interval could be changed on line by the subjects by adjusting a parameter on the computer menu.

Data Acquisition

The subjects were filmed with a 16-mm high-speed motion picture camera (Teledyne type DBM 55, Teledyne Camera Systems, Arcadia, CA) running at a frame rate of 100 Hz. The camera was placed in front of the subjects, at a distance of 6 m. The focus of the zoom lens was adjusted so that the subject and all the ball trajectories were in view of the camera. A flashing light with a fixed frequency of 2.0 Hz was placed right behind the subject so that the nominal and the actual frame rate of the camera could be compared in later analyses. A plumb line suspended from the ceiling defined the gravitational vertical.

After professional development and further processing, the films (Kodak 7251, Ektachrome high-speed daylight film, 400 ASA) were projected onto the opaque screen of a film motion analyzer (NAC type MC OF) by means of a 16-mm projector (NAC type RH 160F), connected to a personal computer. The position of the projector was adjusted to align the plumb line with the vertical axis of the screen. Frame by frame, the x- and y-coordinates of the centers of the balls were digitized, fed into the computer, and stored on floppy disk for later analysis. The actual frame rate of a trial was estimated by averaging the number of frames between the onsets of the flashlight over the digitized trial. The actual frame rates varied between 102.0 Hz and 111.0 Hz over the recorded trials. The actual frame rates provided the correct time basis in the subsequent computation of the duration of the various time components in juggling.

Data Reduction

The following procedure was used to identify the moments of catch and release: First, the raw displacement data were filtered with a recursive second-order Butterworth filter (cut-off frequency: 10 Hz). This procedure was run through forward and backward to eliminate the phase shift (Lees, 1980; Wood, 1982). The filtered displacement data along the y-axis were differentiated to obtain the velocity of the ball in the vertical direction. Subsequently, a peak-finding algorithm, involving extrapolation techniques, was used on the velocity trajectories of the balls to determine the temporal locations of throwing (positive peak velocity) and catching (negative peak velocity).

From these moments in time, the individual values of the following temporal variables were calculated: time flight (t_{f_i}), the time between a throw and a catch of the same ball; time loaded (t_{l_i}), the time between a catch and a successive throw of the same ball; time unloaded (t_{u_i}), the time between a throw and a successive catch performed by the same hand; hand cycle time ($t_{h_i} = t_{l_i} +$

t_{u_i}), and the ratio of time loaded to hand cycle time ($k_i = t_{l_i}/t_{h_i}$). For each recorded trial, the mean values of $t_h (= \Sigma t_{h_i}/n)$ and k ($= \Sigma k_i/n$) were computed ($n = 6$ for the cycle trials and $n = 12$ for the test trials) for later statistical use. The inaccuracy of measuring these variables was one frame at most, which corresponds to maximal errors in estimating time loaded and time unloaded of 1.5 and 4.5%, respectively; due to interdependency of errors this results in a maximal error in the ratio of time loaded to hand cycle time of less than 1%.

Results and Discussion

Cycle Trials

After three practice sessions, at the time of the pretest, all subjects were able to juggle at least one complete cycle. There were, however, considerable individual differences in the amount of progress made up to that point in time, as can be observed from Figure 2A, which shows the amount of practice required for each individual subject to perform n complete juggling cycles (n from 1 to 6). Figure 2B shows the averaged data and the associated between-subject variability. As can be seen, the averaged learning rate was almost linear. On average, subjects could juggle 3.4 complete cycles after the 90 min of practice leading up to the pretest.

To test the hypothesis that in the first stage of learning subjects are striving to satisfy the task constraint imposed by the common equation of juggling, we calculated a compound measure (z_i) for the deviation from this equation for each individual hand/ball cycle (defined as t_{l_i} and the subsequent t_{u_i} and t_{f_i}), according to the formula:

$$z_i = \{[(N/H) - 1](t_{l_i}/t_{f_i})\} + (Nt_{u_i}/Ht_{f_i}) - 1,$$

which can be derived directly from the common equation of juggling $[(t_l + t_u)/H = (t_l + t_f)/N]$. From these individual z_i values, the mean absolute deviation ($|z|$) from the common equation of juggling over a juggling cycle ($n = 6$) was computed by taking

$$|z| = \left|\sum_i^n \frac{z_i}{n}\right|.$$

A repeated-measures ANOVA on the $|z|$ scores of the cycle trials (1 to 6) revealed a significant main effect for the cycle trial number, $F(5, 95) = 5.87$, $p < .0001$. Further analysis of the data revealed that the $|z|$ scores on the cycle trials, which are summarized in Table 1, decreased systematically with increasing performance, indicating that ball cycle time and hand cycle time become better entrained as longer bouts of juggling can be sustained, as was expected from theory.

Table 1 shows that the k values attained during the first cycles of juggling were slightly higher but still remarkably close to .75. A repeated-measures ANOVA on the k scores per cycle showed no significant effect of cycle trial number, $F(5, 95) = 1.33$, $p > .10$, suggesting that k does not change during the early phase of learning. The mean value of all ks observed during the six cycle trials was .76, which proved to be significantly higher than .75 in a two-tailed t test, $t(199) = 3.16$, $p < .005$. The t_h values were close to 1 s and decreased systematically as learning progressed, $F(5, 95) = 3.98$, $p < .005$.

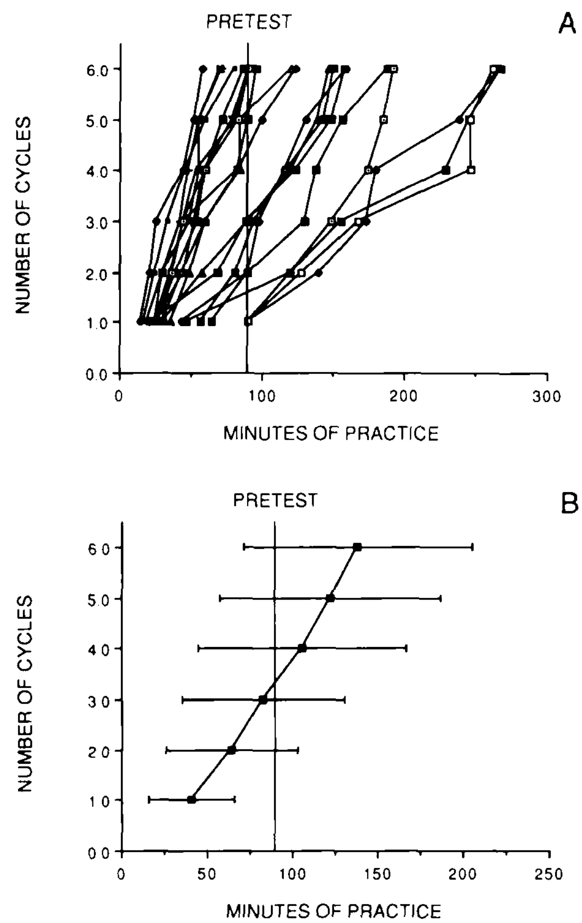

FIGURE 2. (A) Individual learning curves: number of undisturbed juggling cycles versus minutes of practice; (B) Learning curve averaged over subjects, with the between-subject standard deviation indicated by horizontal bars.

Test Trials

Table 2 summarizes the main experimental test results in terms of the mean k and t_h values (and their standard deviations) for the instructor-plus-metronome (IM) group and the instructor-only (IO) group, respectively. As was found in the analysis of the cycle trials, the group means for the k values obtained were in the vicinity of .75 (only the k values achieved by the IM group on the pretest were significantly higher than .75, $t(9) =$

TABLE 1
Means (and Between-Subject Standard Deviations) of | z | (–), k (–), and t_h (s) on the Cycle Trials

	Cycle number					
	1	2	3	4	5	6
| z |	.05 (.04)	.03 (.03)	.02 (.01)	.01 (.01)	.02 (.01)	.01 (.01)
k	.76 (.05)	.76 (.03)	.77 (.03)	.76 (.04)	.76 (.05)	.75 (.05)
t_h	1.08 (.11)	1.05 (.09)	1.03 (.09)	.98 (.11)	1.01 (.14)	.99 (.12)

Note. N = 20 subjects.

TABLE 2
Means (and Between-Subject Standard Deviations) of k and the Absolute Value of t_h (in s) on the Test Trials for the Instructor-Plus-Metronome Group (IM) and the Instructor-Only Group (IO)

	Pretest		Midtest		Posttest	
Group	k	t_h	k	t_h	k	t_h
IM	.78* (.04)	.93 (.13)	.75 (.06)	.90 (.15)	.75 (.04)	.88 (.14)
IO	.77 (.06)	1.08 (.20)	.76 (.04)	.94 (.16)	.74 (.04)	.90 (.16)
M	.77 (.05)	1.00 (.19)	.76 (.05)	.92 (.15)	.74 (.04)	.89 (.15)

Note. Both groups were comprised of 10 subjects.
* = significantly different from .75 at the .05 level on a one-tailed *t* test.

TABLE 3
Means (and Between-Subject Standard Deviations) of k and the Absolute Value of t_h (in s) on the Test Trials for the Faster Learning Group (FL) and the Slower Learning Group (SL)

	Pretest		Midtest		Posttest	
Group	k	t_h	k	t_h	k	t_h
SL	.79* (.04)	1.06 (.18)	.78 (.05)	.98 (.12)	.76 (.03)	.93 (.13)
FL	.75 (.05)	.94 (.18)	.74 (.05)	.85 (.16)	.73* (.04)	.84 (.16)
M	.77 (.05)	1.00 (.19)	.76 (.05)	.92 (.15)	.74 (.04)	.89 (.15)

Note. Both groups were comprised of 10 subjects.
* = significantly different from .75 at the .05 level on a one-tailed *t* test.

1.88, $p < .05$, one-tailed), whereas all group means for the t_h values obtained in the test trials were close to 1 s. Hence, the mean values obtained for the two groups differed very little. Indeed, ANOVAs of the k and t_h values with factors group (IM and IO) and test moment (pretest, midtest, and posttest values), with repeated measures on the last factor, confirmed that the addition of the metronome had not made a significant difference with regard to the k and t_h values obtained, $F(1, 18) = 0.00$ for the k values, and $F(1, 18) = 1.24$, $p > .10$ for the t_h values. The only significant effects found were main effects on k and t_h for the test moments, $F(2, 36) = 4.73$, $p < .02$ for the k values, and $F(2, 36) = 14.84$, $p < .0001$ for the t_h values, indicating that the k values as well as the t_h values decreased systematically for both groups in the course of the experiment.

To gain further insight into the learning process in terms of the time-evolution of the k and t_h values, we regrouped the subjects into a group of faster learners (FL) and a group of slower learners (SL). To this end, we defined the 10 subjects who first reported to have successfully accomplished six complete juggling cycles as *faster*, the other 10 as *slower*. As it turned out, both groups were comprised of 5 subjects from the instructor-plus-metronome group and 5 subjects from the instructor-only group.

The mean k and t_h values for these two new groups are collected in Table 3. From this table it can be seen that, whereas the mean k and t_h values for both the fast- and the slow-learning group decreased systematically in the course of the experiment, the slower learners produced k and t_h values that were higher on average on each test occasion than those attained by faster learners. Repeated-measures ANOVAs with factors group (SL and FL) and test moment confirmed these observations statistically by revealing the following significant main effects: test moment, $F(2, 36) = 4.54$, $p < .02$, for the k values; and $F(2, 36) = 11.60$, $p < .0001$, for the t_h values; group, $F(1, 18) = 4.76$, $p < .05$, for the k values; and $F(1, 18) = 6.22$, $2p < .05$, for the t_h values. Interaction effects were not significant, $ps > .10$.

On average, the SL group juggled at k values larger than .75 and gradually lowered their k values in the direction of $k = .75$, whereas the FL group already juggled at $k = .75$ by the time of the pretest and gradually learned to achieve k values smaller than .75. The k values produced by the slower learners during the three tests were, when taken together, significantly higher than .75, $t(29) = 3.19$, $p < .005$, two-tailed, sample mean $k = .77$; whereas the total set of k values of the faster learners were not significantly different from .75, $t(29) = -1.44$, $p > .10$, two-tailed; sample mean $k = .74$. Importantly, the k values achieved by the SL group on the pretest were significantly higher than .75, $t(9) = 2.72$, $p < .02$, one-tailed; whereas those achieved by the FL group were not. Conversely, on the posttest, the k values produced by the FL group were significantly lower than .75, $t(9) = -1.81$, $p < .05$, one-tailed; whereas those produced by the SL group were not.

Although at the time of the pretest the FL group juggled at $k = .75$, indicating that they had already attained the primary mode-locked solution, analysis of their k values produced on the cycle trials recorded before the pretest (i.e., up to the time they were able to juggle five complete cycles) revealed that they too had started at k values significantly larger than .75, $t(49) = 1.82$, $p < .05$, one-tailed, sample mean $k = .76$.

General Discussion

In the past, acquiring the skill of juggling has been described as a process of (a) "debugging" (Minsky & Papert, 1972), (b) "intellectually" constructing motor programs (Austin, 1976), and (c) discovering specific "grammatical" rules (Norman, 1976; Steiner, 1988). Clearly, what these studies have in common is that the learning process of juggling is viewed as the formation of a cognitive representation, for example, of an abstract control structure.

Although we do not wish to belittle the role of cognition in learning to juggle, the concept of a cognitive representation has drawn attention away from the dynamical principles at work in skill acquisition. Quite a different picture of learning to juggle emerges from the dynamical systems perspective adopted in this study.

In the present investigation, we focused on quantitative, theoretically based descriptions of the three stages of learning to juggle a three-ball cascade pattern. By and large, the results obtained from novices are in agreement with a three-stage learning hypothesis that we formulated on the basis of our insights into the expert juggler's performance (see Figure 3).

In Stage 1 of learning, the novice is busy actively discovering the real-time requirements of juggling as expressed in the common equation of juggling. Hands and balls come to move in the same time frame, and the basic challenge confronted is to ensure that the events associated with the hands match the events associated with the balls in time (e.g., catch when there is a ball to catch). Once this has been achieved, the first few juggling cycles can be performed, allowing the novice to begin to explore how the internal timing of a juggling hand cycle should be partitioned to obtain a stable performance (Stage 2).

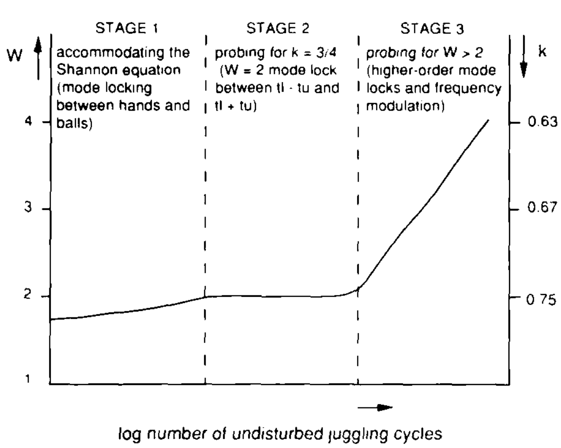

FIGURE 3. Theoretical picture of the three phases of learning the cascade juggle.

The start of Stage 2 of discovering the primary fixed point of $k = .75$ may be defined operationally when the k variable begins to drift. As we have seen, the k values attained in the first few complete juggling cycles are somewhat higher than .75, indicating that the winding number for the internal organization of the juggling hand cycle is still below its first possible locking value ($W = 2$), although very close to it. This fact, in itself, shows how intimately the value of $k = .75$ is tied to the temporal order of juggling required to sustain the act for any length of time, as internal mode locking between the repetitive subtasks of the hand lends stability to the system. The importance hereof could be further substantiated by analyzing the subtle but important differences in k that were found between faster and slower learners. The slower learners (SL) produced k values that were slightly but significantly higher than .75 (ca. .77) and had considerable difficulty in braiding the singular events of catching and throwing of individual balls together into a continuous flow. Their learning process may be viewed as one of finding the *mode-locked solution* of $k = .75$. In contrast, the faster learners (FL) had already produced k values significantly higher than k before that time. The faster learners apparently had no great difficulty achieving a smooth performance, as indexed by the amount of practice time required to juggle for a certain length of time without disruptions. Their efforts during the last seven sessions may therefore be characterized as finding the principles of *frequency modulation* around the symmetry solution of $k = .75$, allowing them to attain smaller k values than $k = .75$ without jeopardizing the integrity of their performance. Hence, the FL group quickly entered learning Stage 3.

Stage 3 of learning is characterized by frequency modulation away from the mode-locked value of .75. The degree to which the FL jugglers in the present experiment managed to achieve this, however, was still very small, especially when their performance is compared to expert jugglers, who prefer to juggle three balls at frequencies much higher than 1 Hz (ca. 2.5 Hz) and k values considerably lower than .75. Experts operate around other fixed points in the workspace, such as $k = 2/3 = .67$ ($W = 3$) and $k = 5/8 = .63$ ($W = 4$). Apparently, (much) more learning time is required than 10 times 1/2 h to achieve this. The important conceptual point, however, is that faster learners, just by virtue of the fact that they have discovered the fixed point of $k =$

.75, have acquired the ability to modulate, explore around the locking mode at $k = .75$, without losing (too much) stability. As argued in Beek (1989b), it seems to be a hallmark property of skillful biological systems that they operate close to mode-locking regimes without entering into them. Such systems seek the best of two worlds: stability and reliability on the one hand (mode locking) and adaptability and flexibility on the other (modulation). Recently, this notion has received further experimental and theoretical attention (DeGuzman & Kelso, 1991; Kelso, DeGuzman, & Holroyd, 1991). The implication for a theory of learning complex movement skills such as juggling is that the discovery of fixed points, relative to which modulation and transient behavior can take place, logically preceded the acquisition of the ability to modulate around the mode-locked regimes, as is reflected in our second and third stage of learning. Behavioral flexibility in a wide range of operations with multiple stable fixed points to resort to when necessary seems to be a defining characteristic of expertise in complex movement systems.

In sum, the findings reported here support the general hypothesis that attaining the temporal solution of $k = .75$ is indeed an essential milestone to be overcome in learning how to juggle, and, in so doing, lend credibility to the analytic model presented in the Appendix. Especially, the sensitivity of the k measures, as reflected in small differences that, nevertheless, produced strong statistical effects, underscores the appropriateness of k as an ensemble variable to describe the temporal order of (cascade) juggling.

This leaves us with the question why the presence of a metronome, in addition to an instructor, proved to be of no help in learning the three-ball cascade. After all, given the fact that the problem of learning to cascade juggle is first and foremost a problem of mastering timing rleations, the expectation that novice jugglers would benefit from an external pacing signal was plausible. Two explanations come to mind as to why this expectation was not borne out. One explanation might be that the signal provided did not specify the required temporal structuring in sufficient detail. As intimated, the beep-beep interval only specified the rate of juggling and not the internal partitioning of the overall hand cycle durations in exact values of t_l and t_u, and, a forteriori, t_f. It contained no information about the value of $k = .75$. Such would have been the case only if, in addition to the beep-beep interval, a particular corresponding height criterion relative to the hands (e.g., in the form of a visual target) had been specified to the juggler. A second, more general explanation might be that learning to juggle is very much a closed process. By this we mean that the nature of the juggling workspace might be such that it is difficult to find a feedback form that can penetrate that space. The metronome manipulation rests on the assumption that there exists a specific mapping from the external time signal to the internal temporal structuring of the component activities of juggling itself to which the juggler is sensitive. Perhaps this was not the case.

A key assumption of the dynamical systems approach to movement coordination is that perceptual–motor tasks may be characterized by the existence of one or a few fixed points (such as $k = .75$) in the total set of values that the ensemble variable(s) for the task in question may attain. What we have demonstrated in this article is that prior identification of these ensemble variable(s) and their invariant and variant properties in expert performers may provide a frame of reference with the help of which stages in a learning process may be distinguished and studied. Hence, it is also possible to study learning processes from a dynamical point of view in cases in which we do not know the initial state.

From the point of view of dynamical systems theory, the challenge is to provide a principled, law-based, account of the acquisition of a new skill in terms of the dynamics of the developing workspace. The research reported in this article was directed at defining the prerequisites of such an account. Empirical evidence was provided for the first two of the three stages expected in the development of the novice juggler's performance, although the data contained glimpses of the third stage. To fully test this hypothetical third stage of learning (i.e., from $W = 2$ up to $W = 12$), however, we need to perform experiments in which subjects are followed for much longer learning times. Moreover, to exactly model the dynamics of the changing attractors layout all the way from the initial state of the novice to the considerably richer end state of the expert, we require fine-tuned analyses of the behavior of k and its range of operation on a cycle-to-cycle basis.

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APPENDIX

A more elaborate model for the internal partitioning of a juggling hand cycle and frequency locking between its dynamical components may be derived through an analysis based on what is called the Mathieu-Hill equation (see Whittaker & Watson, 1962)

$$\ddot{x} + \omega^2 \{1 + \lambda(t)\}x = 0.$$

The essence of this equation is that it provides an expression for an oscillating system in which frequency and phase modulation may occur because of the presence of the time-dependent modulation term $\lambda(t)$, which, in a number of generalized cases, can be identified as a Lagrange multiplier. If $\lambda(t) = 0$, the motion of the oscillator is perfectly harmonic. If $\lambda(t) \neq 0$, the motion will be near- (or pseudo-) harmonic, or even erratic (when $\lambda[t]$ is large and stands in particular relation to ω). Thus, only by manipulating the modulation term, a series of different dynamical regimes may be induced. This property makes the Mathieu-Hill equation a generic one. It provides a reasonable model for an oscillating hand because rhythmic movements are usually frequency modulated to some degree.

Let us now elaborate this picture of a frequency-modulated hand for the skill of cascade juggling. The basic structure for the law describing the temporal microstructure of the partitioning of the hand cyle time (t_h) in the time that a hand is filled with a ball (t_l) and the time that the hand is empty (t_u) is

$$\frac{d^2x}{d(\omega_h t)^2} + \{1 + A\cos a\omega_h t\}x = 0,\ 0 < t < t_l$$

where x represents the horizontal position of the oscillating hand, A is the amplitude of the modulation term, a is the modulation parameter (determining how many times the modulation function passes through zero during the hand cycle), and $\omega_h = 2\pi/t_h$.

To ensure that this modulated behavior of a hand moving with a ball goes, at ball release, into a stable harmonic mode, the value of the modulation function has to go to zero at this point in the hand loop. If n is the number of consecutive throws after which the initial situation (i.e., ball release) is regained in full, then this requirement can be met by obeying the rule

$$a(A)\omega_h t_l = (\pi/2) + n\pi,$$

or

$$a(A)(t_l/t_h) = (1/4) + (n/2), \text{ with } (t_l/t_h) < 1,$$

with $a(A)$ being rational.

The preceding equations reveal that when modulation is to be restricted, discrete ratios between characteristic time scales have to be adhered to, resulting in specific ratios of t_l/t_h. For the most basic conditions for phase locking between the angular frequency of modulation and the angular frequency of the hand, $a(A) = 1$ and $n = 1$, $t_l/t_h = 3/4$—implying that stable, precise throwing is characterized by holding onto a juggled object for 3/4 of the cycle time of the hand.

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